Testing for Equivalence

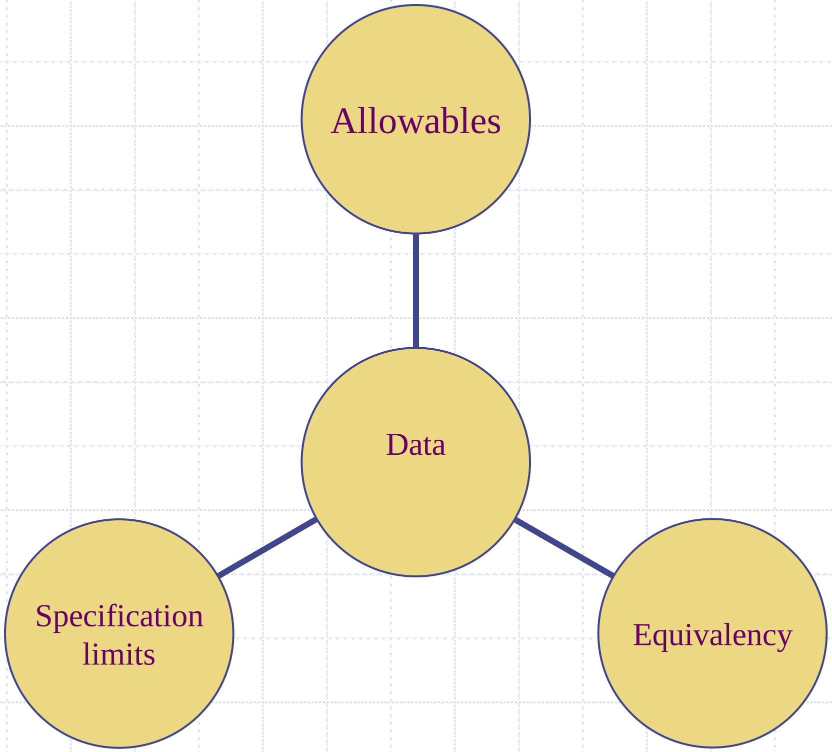
- Purpose of Equivalence
- CMH17 Statistical Analysis Methods

Purpose of Equivalence

- To demonstrate material equivalency: to assure that “minor” changes to the material and/or process do not adversely affect material properties. Recommended alpha = 5%, n ≥ 8
  - Examples: New part fabricator, new resin mixer, new prepreg line, new raw ingredient supplier, etc.
- To establish specification limits: To avoid using material specification limits that are based on application requirement (performance-based specification) or other arbitrary methods that are not based on actual material property or statistics. Recommended alpha = 1%, n=5.
- To avoid using A-basis and B-basis values as material specification limits – basis values have no statistical relationship to quality control
- Published in DOT/FAA/AR-03/19 and MIL-HDBK-17-1F section 8.4.1

Linkage Between Allowables, Equivalency, and Specification Limits

- The same dataset is used to derive basis values, equivalency limits, and specification limits (i.e. common mean and standard deviation).
- Equivalency and quality control tests are designed to protect material allowables.



Specification Development Guidelines

- DOT/FAA/AR-07/3 Guidelines and Recommended Criteria for the Development of a Material Specification for Carbon Fiber/Epoxy Unidirectional Prepregs Update
- DOT/FAA/AR-06/10 Guidelines and Recommended Criteria for the Development of a Material Specification for Carbon Fiber/Epoxy Fabric Prepregs
- DOT/FAA/AR-02/110 Guidelines for the Development of Process Specifications, Instructions, and Controls for the Fabrication of Fiber-Reinforced Polymer Composites

Sample Equivalency Test Matrix

Layup	Test Type and Direction	Property	Number of Batches x Number of Panels x Number of Test Specimens Test Temperature/Moisture Condition			
			CTD	RTD	ETD	ETW
$[0]_{4S}$	ASTM D3039 Warp Tension	Strength + Modulus	1x2x4	1x2x4		1x2x4
$[0]_{4S}$	ASTM D6641 Warp Compression	Strength + Modulus		1x2x4		1x2x4
$[90]_{4S}$	ASTM D3039 Fill Tension	Strength + Modulus	1x2x4	1x2x4		1x2x4
$[90]_{4S}$	ASTM D6641 Fill Compression	Strength + Modulus		1x2x4	1x2x4	1x2x4
$[45/-45]_{2S}$	ASTM D3518 In-Plane Shear	Strength + Modulus	1x2x4	1x2x4		1x2x4
$[0]_{17}$	ASTM D2344 Short Beam Strength	Strength		1x2x4		1x2x4
(25/50/25 - QI) $[45/0/-45/90]_S$	ASTM D5766 Open Hole Tension	Strength	1x2x4	1x2x4		1x2x4
(25/50/25 - QI) $[45/0/-45/90/-45/90]_S$	ASTM D6484 Open Hole Compression	Strength		1x2x4		1x1x4
(25/50/25 - QI) $[45/0/-45/90/-45/90]_S$	ASTM D7136 & D7137 Compression after impact 1500 in-lbs/in	Strength		1x1x6		

Equivalency Testing

- Testing is not as extensive as that needed to set basis values, so fewer tests are run at fewer environmental conditions and fewer specimens/batches are required.
- *Test matrix* must specify which tests will be performed and under what environmental conditions; must be decided based on the purpose of the equivalency
- The stakeholders must agree on the *test matrix* specifying exactly what tests will be done at what environmental conditions and how many specimens from how many batches.

Statistical Terminology

- Hypothesis Testing
- Type I and Type II errors
- Test Statistics
- Coefficient of Variation (CoV)

Hypothesis Testing

- We check each characteristic tested but what constitutes a ‘failure’ depends on what is being tested.
  - For Strength values, the test statistics are only concerned if they are lower than the original qualification material (although significantly higher values could be undesirable). We test both the sample mean and the sample minimum individuals against thresholds computed from the qualification sample.
  - For Modulus values, a failure occurs if the equivalence sample mean is either too high or too low. An acceptable interval is computed from the qualification sample.

Hypothesis Testing

- Null Hypothesis: (H_0)
  - No Difference. There are no significant differences between the two materials on the characteristic being tested
- Alternate Hypothesis (H_1)
  - The two materials are different with respect to the characteristic tested
- The null hypothesis is assumed true and we examine our sample data to see if it is consistent with this assumption. If not, we reject the null hypothesis and the equivalence sample fails that test.

Type I and Type II Errors

	Conclude that material is equivalent	Conclude that material is not equivalent
Material is equivalent	No Error	Type I Error (α) wrongly assume material is not equivalent.
Material is not equivalent	Type II Error (β) Wrongly assume material is equivalent	No Error

Recommendation: α = .05 for equivalence testing and α = .01 for acceptance testing.

Test Statistic for Modulus

$$t_0 = \frac{\overline{X}_{\text{Qual}} - \overline{X}_{\text{Equiv}}}{S_{\text{pooled}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- This statistic is compared to the critical value of the t-distribution with the appropriate degrees of freedom at the 95% confidence level for equivalence testing and at the 99% confidence level for acceptance testing.
- If the t-statistic is larger than the critical value, the null hypothesis of equality is rejected and the material fails the equivalence test.

Test Statistics for Strength

- Failure for decrease in mean or minimum individual
  - Failure for decrease in mean

$$W_{mean} = \bar{x} - k_n^{mean} \cdot S$$

  - Failure for decrease in minimum individual

$$W_{min\ \ indiv} = \bar{x} - k_n^{min\ indiv} \cdot S$$

  - If the equivalence sample fails either of these two comparisons, it fails the test.
  - The k_n are found in DOT/FAA/AR-03/19 tables 20 and 21.
  - If the two materials are equivalent, the probability of failing either one or both of the tests is α.

Co-efficient of Variation

The Co-efficient of Variation (CoV) is the standard deviation divided by the mean.

$$CoV = \frac{S}{\bar{x}}$$

- If CoV is erroneously high – we will accept too many “bad” materials
- If CoV is erroneously low – we will reject too many “good” materials

Modified CoV Approach

- Original material qualification or allowables program often fail to capture the true material property variability. The "as-measured" CoVs are often lower than actual CoV.

- Modified CoV adjusts the CoV higher for a temporary period of time to account of variability not captured in the original program.
  - Modified CoV might make basis values lower
  - Modified CoV also might make material equivalency/acceptance limits lower

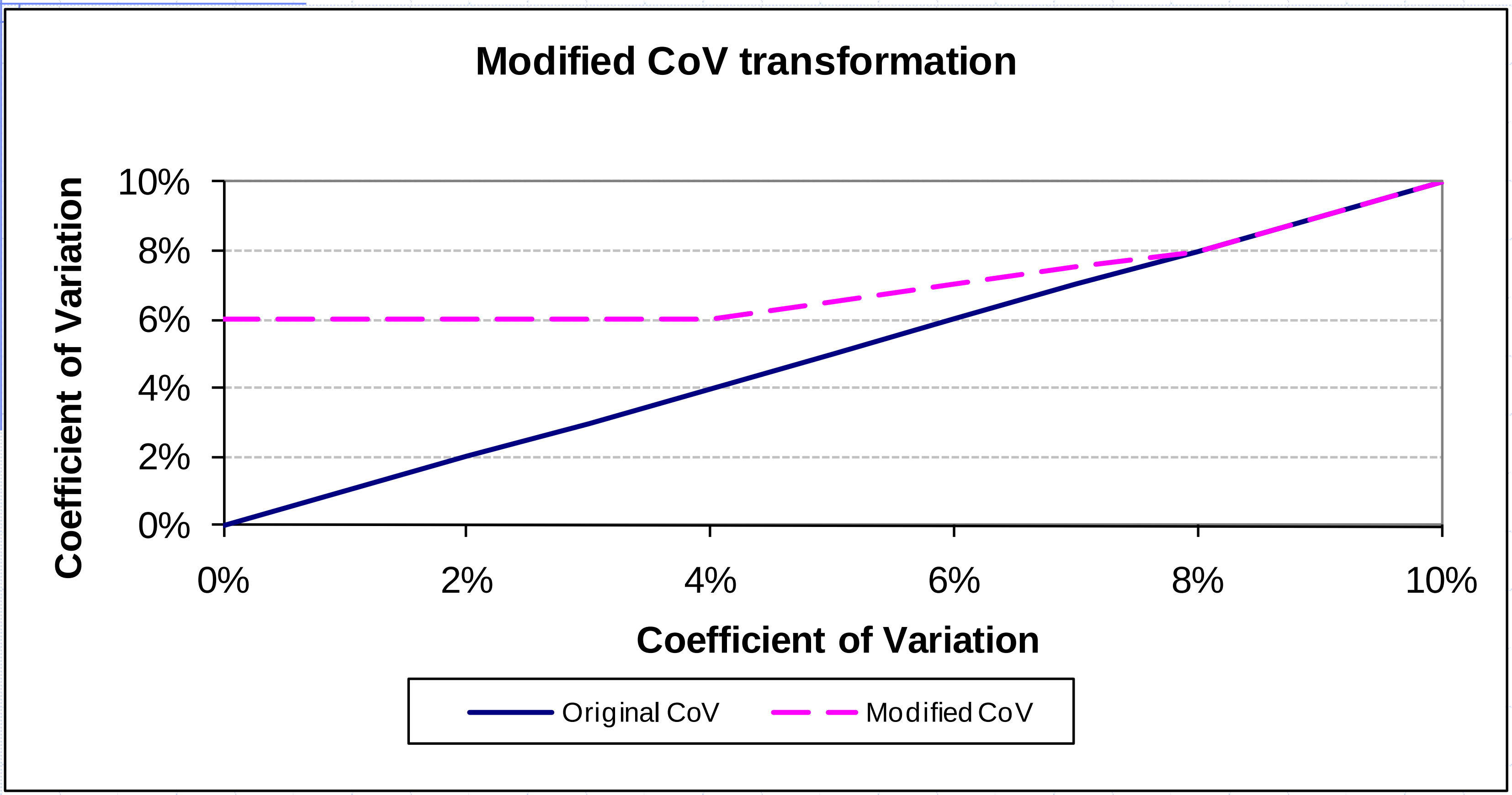
- If Modified CoV is used for equivalency or material acceptance, it must also be used for the calculation of material allowables.

Effects of Modified CoV (in statistical point of view only)

- In order to compensate for the additional variation (not present in the current dataset), the CoV is increased PRIOR to computing the A and B basis values.

- The effect of this will be:
  - Slight decrease in the computed A-basis and B-basis values
  - Slight decrease in the number of failures during equivalence or acceptance testing.

Graph of Modified CoV

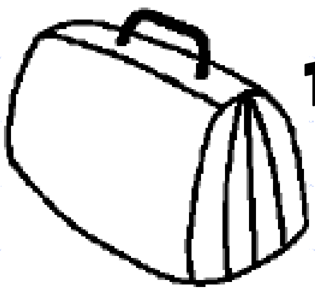


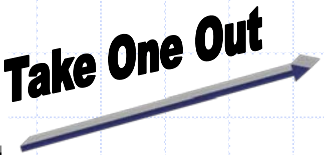
CoV Modification Rules

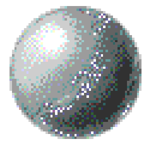
- If as-measured CoV < 4%:
  - Modified CoV = 6%
- If as-measured CoV is between 4% and 8%, adjust upward according to the following formula:
  - Modified CoV = ½ as-measured CoV + 4%
- If as-measured CoV is between 8% and 10%:
  - Modified CoV = as-measured CoV
- If as-measured CoV > 10%:
  - Modified CoV = 10% for equivalency & acceptance purposes
  - Use as-measured CoV for calculating basis values

What Happens to α During a Retest

A truck has 100 bags; one of them is a white color bag (α=0.01)







The white color bag has 100 marbles; one of them is a white color marble

- The probability of taking the white color bag from the truck and taking the white color marble from the white color bag is 0.0001 (Multiplication Rule).
- For initial α=0.01, retest α=0.01*0.01=0.0001=0.01%
- For initial α=0.05, retest α=0.05*0.05=0.0025=0.25%

Decision Making Process

- We cannot expect zero failures even when the two materials are identical. The probability of 1 or more failures is computed using the binomial distribution.
  - Example1 :   The probability of 0 failures in 25 tests when the samples actually are equivalent is $.95^{25} = .2775$. The probability of 1 or more failures due to chance alone is the complement:

    $$1 - .2774 = .7226$$

  - Example 2: A recent equivalence comparison was done with 78 separate tests; if the two materials are equivalent, the probability of 0 failures due would be $.95^{78} = 0.0183$. Thus, the probability of 1 or more failures is:

    $$1 - .0183 = .9817$$

Decision Making Process

- How do we decide whether or not a material is equivalent if we expect some failures? How many is too many?
  - Currently there is no firm cut-off point. All failures must be investigated as to the cause and engineering judgment used to make the determination.
  - Determination should be based on the number of failures as well as the severity of each failure.

Engineering Judgment Guidelines

- If there are only a few failures, then retests can be done. If the material passes the retest, we can presume the failure was due to random chance rather than a true material difference.
- If the pattern of test results show a consistent problem – i.e. retests show the same problem or the failures consistently occur for the same characteristic, then the material is NOT equivalent.

Example: retests needed

NCAMP: Equivalency Test Results

Test	Type	Environ-ment	Results
Warp Tension	Strength	CTD	Pass
		RTD	Pass
	Modulus	CTD	Pass
		RTD	Fail
Fill Tension	Strength	CTD	Pass
		RTD	Pass
		ETW (200)	Pass
	Modulus	CTD	Pass (mod CV)
		RTD	Pass
		ETW (200)	Pass
Fill Compression	Strength	RTD	Pass
		ETD	Pass (mod CV)
		ETW (200)	Pass
	Modulus	RTD	Pass
		ETD	Pass
		ETW (200)	Pass

Test	Type	Environ -ment	Results
Warp Compression	Strength	RTD	Pass
		ETW (200)	Fail
	Modulus	RTD	Pass
		ETW (200)	Pass
Short Beam Shear	Strength	RTD	Pass
		ETW (200)	Pass
In Plane Shear	0.2% Strength	RTD	Pass
		CTD	Pass
	5% Strength	RTD	Pass
		CTD	Pass
	Modulus	RTD	Fail
		CTD	Pass (mod CV)
Open Hole Compression	Strength	RTD	Pass
		ETW (200)	Pass
Open Hole Tension	Strength	RTD	Pass
		CTD	Pass

Example: Fails equivalency

NCAMP: Equivalency Test Results

Test	Type	Environment		Alt Cure Cycle	Alt Cure Cycle - Modified CoV
Warp Compression	Strength	RTD	Mean	FAILS EQUIVALENCY	FAILS EQUIVALENCY
			Minimum	FAILS EQUIVALENCY	PASSES EQUIVALENCY
		ETW (200)	Mean	FAILS EQUIVALENCY	FAILS EQUIVALENCY
			Minimum	FAILS EQUIVALENCY	FAILS EQUIVALENCY
		ETW (250)	Mean	FAILS EQUIVALENCY	FAILS EQUIVALENCY
			Minimum	FAILS EQUIVALENCY	FAILS EQUIVALENCY
	Modulus	RTD		FAILS EQUIVALENCY	PASSES EQUIVALENCY
		ETW (200)		PASSES EQUIVALENCY	PASSES EQUIVALENCY
		ETW (250)		PASSES EQUIVALENCY	PASSES EQUIVALENCY

HYTEQ

- **Hy**pothesis **T**esting of **Eq**uivalence
- NCAMP is introducing this new spreadsheet. It is a template for EXCEL that allows users to input the statistics from each test for the equivalence and qualification samples and the program will compute whether they are equivalent on a test by test basis.
- Engineering judgment still required to determine whether the material passes or fails.

HYTEQ EXAMPLE

Warp Tension Strength & Modulus	Calculated C.V.						Modified CV		
CARBON FABRIC PREPREG	CTD 0° Tension		RTD 0° Tension		ETW 0° Tension (250° F)		CTD 0° Tension	RTD 0° Tension	ETW 0° Tension (250°F)
	MH	LH	MH	LH	MH	LH	Using Modified CV	Using Modified CV	Using Modified CV
As Measured									
Mean Strength (ksi)	135.472	136.680	139.017	138.251	130.154	131.166	135.472	139.017	130.154
Standard Deviation	7.3767	4.3200	7.2198	4.2141	3.5307	3.9332	9.1072	9.1706	7.8093
Coefficient of Variation (CV)	5.4452	3.1607	5.1935	3.0482	2.7127	2.9986	6.7226	6.5967	6.0000
Minimum	125.145	131.751	127.286	130.586	122.682	124.759			
Maximum	146.911	144.070	150.242	146.02	136.463	137.49			
Number of Specimens	19	8	20	8	21	15			
Pooled CV (see Note 2)	4.4076		4.4076		4.4076				
Wmean for α=0.05	130.4631		134.1150		128.3927		129.2881	132.7904	126.2577
Mean Result for α=0.05	PASSES EQUIVALENCY		PASSES EQUIVALENCY		PASSES EQUIVALENCY		PASSES EQUIVALENCY	PASSES EQUIVALENCY	PASSES EQUIVALENCY
Wmin_indiv for α=0.05	115.555		119.524		119.875		110.8823	114.2566	107.4178
Min_indiv. Result for α=0.05	PASSES EQUIVALENCY		PASSES EQUIVALENCY		PASSES EQUIVALENCY		PASSES EQUIVALENCY	PASSES EQUIVALENCY	PASSES EQUIVALENCY
Wmean for α=0.01	128.789		132.477		127.814		127.2216	130.7096	124.9770
Mean Result for α=0.01	PASSES QC ACCEPTANCE TESTING		PASSES QC ACCEPTANCE TESTING		PASSES QC ACCEPTANCE TESTING		PASSES QC ACCEPTANCE TESTING	PASSES QC ACCEPTANCE TESTING	PASSES QC ACCEPTANCE TESTING
Wmin_indiv for α=0.01	111.753		115.803		118.165		106.1894	109.5310	103.6350
Min_indiv. Result for α=0.01	PASSES QC ACCEPTANCE TESTING		PASSES QC ACCEPTANCE TESTING		PASSES QC ACCEPTANCE TESTING		PASSES QC ACCEPTANCE TESTING	PASSES QC ACCEPTANCE TESTING	PASSES QC ACCEPTANCE TESTING
Explanation/Comments									