Statistics for Allowable Generation

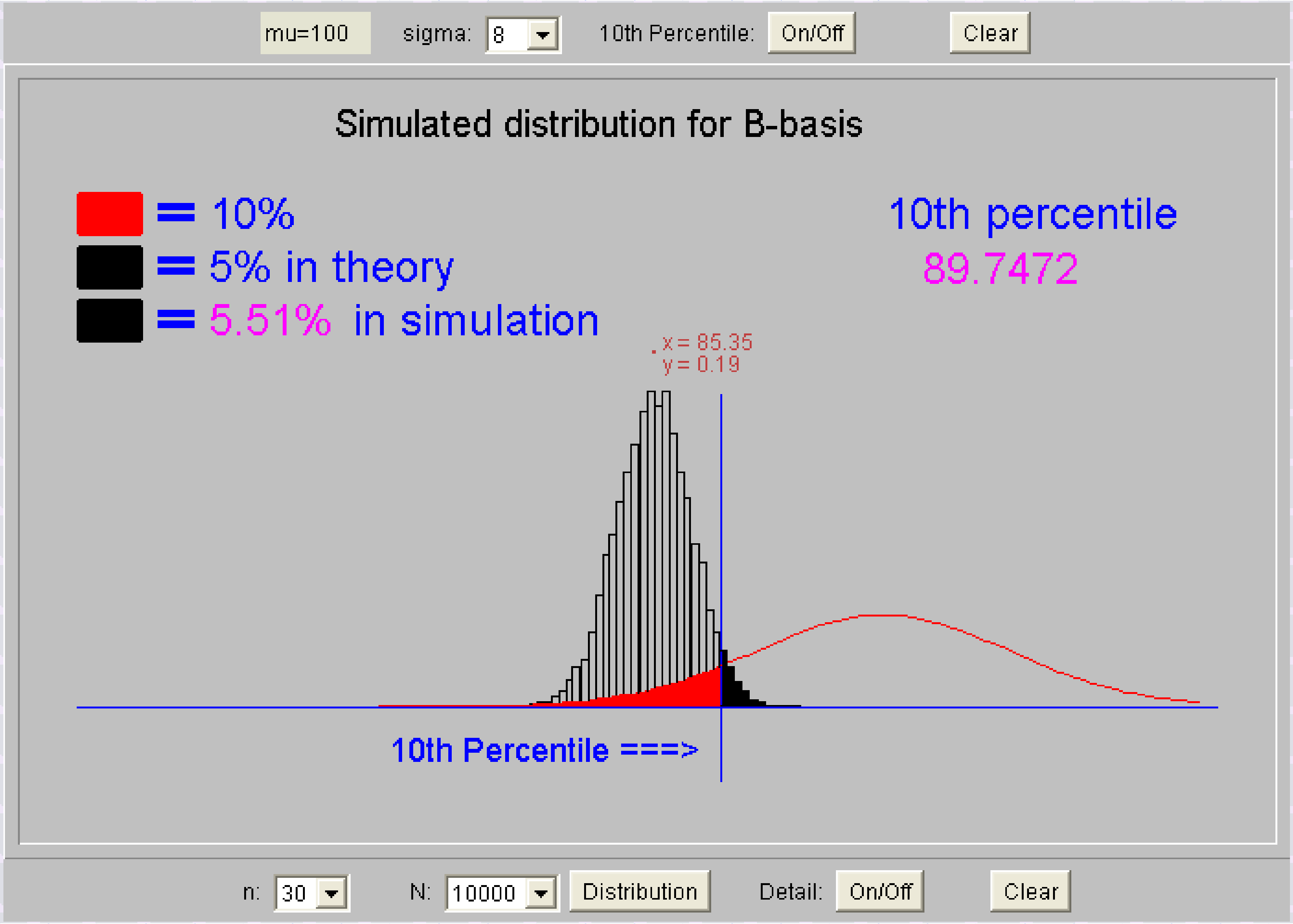
- Statistically Based Material Allowables (Not Design Values)
- CMH-17 statistical analysis methods

A- and B-basis Values

- Design values must be chosen to minimize the probability of structural failure due to material variability. Compliance is typically shown by selecting design values that ensure material strength with the following probability:
  - Where applied loads are eventually distributed through a single member within an assembly, the failure of which would result in loss of structural integrity of the component; 99 percent probability with 95 percent confidence (that is, A-basis value).
  - For redundant structure, in which the failure of individual elements would result in applied loads being safely distributed to other load carrying members; 90 percent probability with 95 percent confidence (that is, B-basis values).

$$A-Basis\,value=\bar{x}-(K_A)\cdot s$$

$$B-Basis\,value=\bar{x}-(K_B)\cdot s$$



The internet browser-based simulation program is available at NCAMP website
http://www.niar.wichita.edu/coe/ncamp_media.asp

CMH-17 statistical analysis methods

- Definitions
- Normalization
- Probability Distributions
- Methods to compute basis values
- Selection of method

Definitions

- Mean – the average (the sum of all values divided by the number of values)
- Standard Deviation – an average of the deviation of each value from the mean
- Co-efficient of Variation – The ratio of the standard deviation to the mean
- Outliers – individual values that are significantly different from the remaining dataset

Definitions (con't)

- Structured versus Unstructured Data
  - Structured data has natural groupings that differ discernibly from each other.
  - Unstructured data has no natural groupings OR no discernable differences between groups.
  - Example of natural groupings:
    - Batches
    - Environmental Conditions
- Pooling Data
  - Combining data from different groups together. This is only allowable if there are no significant differences between the groups.

Normalization

- Mechanical properties that are dominated by the properties of the reinforcing fiber are dependent on the volume fraction of fiber in the laminate
- Assumption: fiber-dominated strength and stiffness properties vary linearly with fiber volume fraction
- MIL-HDBK-17F section 2.3.4.2 provides several data normalization methods that are based on fiber volume fraction, fiber areal weight, and cured ply thickness.

Data Reduction & Summary Sheet

- The data is reduced to a set of statistics that describe the results of each test:
  - Mean
  - Standard Deviation
  - Co-efficient of variation
  - Maximum
  - Minimum
  - Number of Specimens
- For fiber dependent/dominated properties, the results are also normalized and statistics are computed for the normalized results

Example Summary Spreadsheet

Fill Tension Properties (FT) -- (CTD)
Strength & Modulus
Plain Weave Fabric

normalizing t_{ply} [in]: 0.0079

Specimen Number	Batch #	Cure Cycle	Prepreg Lot #	Cure Cycle #	Strength [ksi]	Modulus [Msi]	Avg. Specimen Thickn. [in]	# Plies in Laminate	Failure Mode	Avg. t_{ply} [in]	$Strength_{norm}$ [ksi]	$Modulus_{norm}$ [Msi]
A0NUA115B	A	MH1	1	1	122.999	8.875	0.111	14	LWB	0.0080	123.907	8.941
A0NUA116B	A	MH1	1	1	125.385	9.057	0.111	14	LGM	0.0079	125.631	9.075
A0NUA117B	A	MH1	1	1	118.716	8.660	0.112	14	LAB	0.0080	120.702	8.805
A0NUA211B	A	MH2	1	2	123.009	9.606	0.107	14	LGM	0.0077	119.227	9.311
A0NUA212B	A	MH2	1	2	122.642	9.547	0.108	14	LAB	0.0077	119.519	9.304
A0NUA213B	A	MH2	1	2	120.046	9.172	0.110	14	LAT	0.0079	119.449	9.126
A0NUB115B	B	MH1	2	1	118.887	9.270	0.112	14	LGM	0.0080	119.909	9.350
A0NUB116B	B	MH1	2	1	132.469	9.408	0.111	14	LGM	0.0079	132.988	9.445
A0NUB117B	B	MH1	2	1	133.995	9.326	0.110	14	LGM/LWT	0.0078	133.107	9.264
A0NUB211B	B	MH2	2	2	140.898	10.023	0.102	14	LWT/LWB	0.0073	130.473	9.281
A0NUB212B	B	MH2	2	2	130.999	9.635	0.107	14	LGM/LWT	0.0077	126.933	9.336
A0NUB213B	B	MH2	2	2	124.684	9.459	0.110	14	LGM	0.0078	123.838	9.395
A0NUC115B	C	MH1	3	1	129.872	8.683	0.111	14	LAB	0.0080	130.792	8.744
A0NUC116B	C	MH1	3	1	117.189	8.618	0.112	14	LGM	0.0080	118.178	8.691
A0NUC117B	C	MH1	3	1	130.204	8.528	0.112	14	LGM	0.0080	131.283	8.599
A0NUC211B	C	MH2	3	2	136.238	9.199	0.106	14	LWT/LWB	0.0076	130.962	8.843
A0NUC212B	C	MH2	3	2	129.338	9.146	0.108	14	LGM/LWT	0.0077	126.219	8.925
A0NUC213B	C	MH2	3	2	129.552	8.933	0.110	14	LGM/LWT	0.0078	128.381	8.852
				Average	**127.062**	**9.175**			**$Average_{norm}$**	**0.0078**	**125.639**	**9.071**
				Standard Dev.	**6.605**	**0.407**			**$Standard Dev._{norm}$**		**5.232**	**0.274**
				Coeff. of Var. [%]	**5.198**	**4.441**			**Coeff. of Var. $[\%]_{norm}$**		**4.165**	**3.015**
				Min.	**117.189**	**8.528**			**Min.**	**0.0073**	**118.178**	**8.599**
				Max.	**140.898**	**10.023**			**Max.**	**0.0080**	**133.107**	**9.445**
				Number of Spec.	**18**	**18**			**Number of Spec.**		**18**	**18**

Probability Distributions

- The CMH-17 preferred distribution assumption is normal. This assumption is tested and the basis values are computed by this method in both ASAP and STAT-17.
- If non-normality is observed, then the data can be checked to determine if it fits the Weibull or lognormal distributions and compute the basis values according if one of those distributions is a reasonable fit for the data. These methods are available in STAT-17 only.
- If none of those distributions are a good fit for the data, then the non-parametric method must be used. (Non-parametric technique makes no assumptions regarding the data distribution).

Diagnostic Tests

- Levene's test for equality of variance
  - If data fails Levene's test, then single point analysis must be used
- k-sample Anderson-Darling test for batch equivalence (ADK test)
  - If data fails the ADK test, then ANOVA must be used
- Tests for goodness of fit to
  - Normal Distribution
  - Weibull Distribution
  - Lognormal Distribution
  - If none fits, use non-parametric procedures

Methods to Calculate Basis Values

- ASAP Excel Spreadsheet Macro (K. Suresh Raju, Wichita State University)
  - Pools across environments
  - Assumes pooled data have a normal distribution
  - Assumes different environments have equal variance
- STAT-17 (a.k.a. Single Point) Excel Spreadsheet Macro (J.Adelmann, Sikorsky Aircraft)
  - Will compute for the Normal, Weibull, Lognormal distributions
  - Will compute using Non-Parametric and ANOVA methods
  - Batches pooled within environment
- RECIPE (Ref. M.G.Vangel, A User's Guide to Recipe, NIST, 1994)
  - Normal Distribution only
  - Regression model
  - FORTRAN program

Selecting a method to compute basis values

- In order to pool across environments (i.e. use ASAP software), check that
  - Data within each environment can be pooled across the batches using the Anderson-Darling k-sample test on each environment
  - Pooled dataset is sufficiently normal after standardizing the data. This is checked using the Anderson-Darling test for normality
  - Variances are sufficiently similar after standardizing the data. This is checked using Levene's test.
- Data is standardized by dividing each value by the mean for that environment. This creates datasets which all have a mean of 1.0 and with a standard deviation equal to the co-efficient of variation of the untransformed dataset

Selecting a method to compute basis values

- If pooling across environments is not appropriate, then the basis values for each environment must be computed individually using Single Point method (STAT-17 software).
- If the data from the different batches can be pooled, then the data is checked for goodness-of-fit to the following distributions:
  - Normal Distribution
  - Weibull Distribution
  - Lognormal Distribution
  - If none of these distributions is adequate, then non-parametric method is used
- If the data from the different batches cannot be pooled, the ANOVA method is the only option.

Modified CV Approach

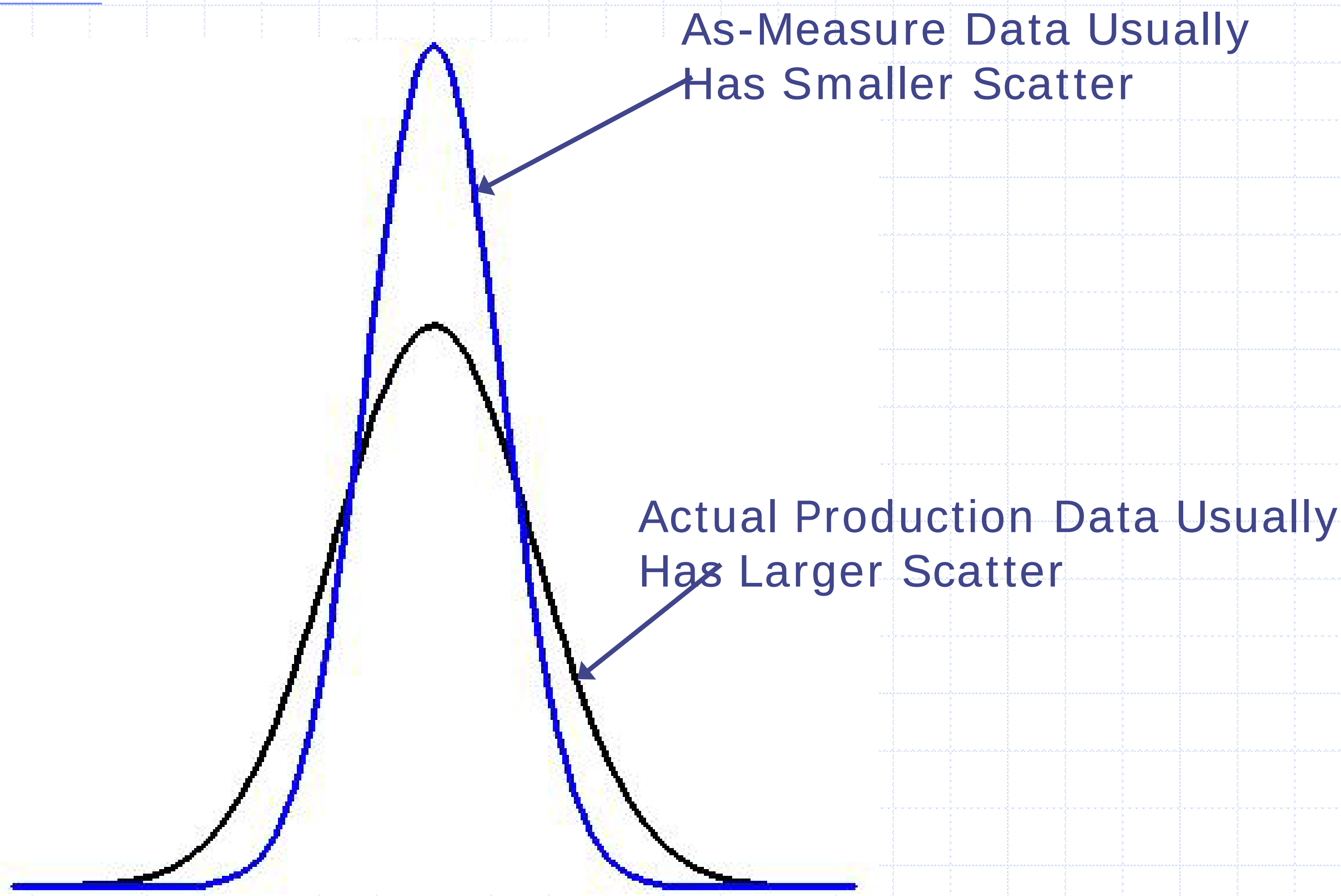
Section 8.4.4 Volume 1 Rev G

(currently available in ASAP only)

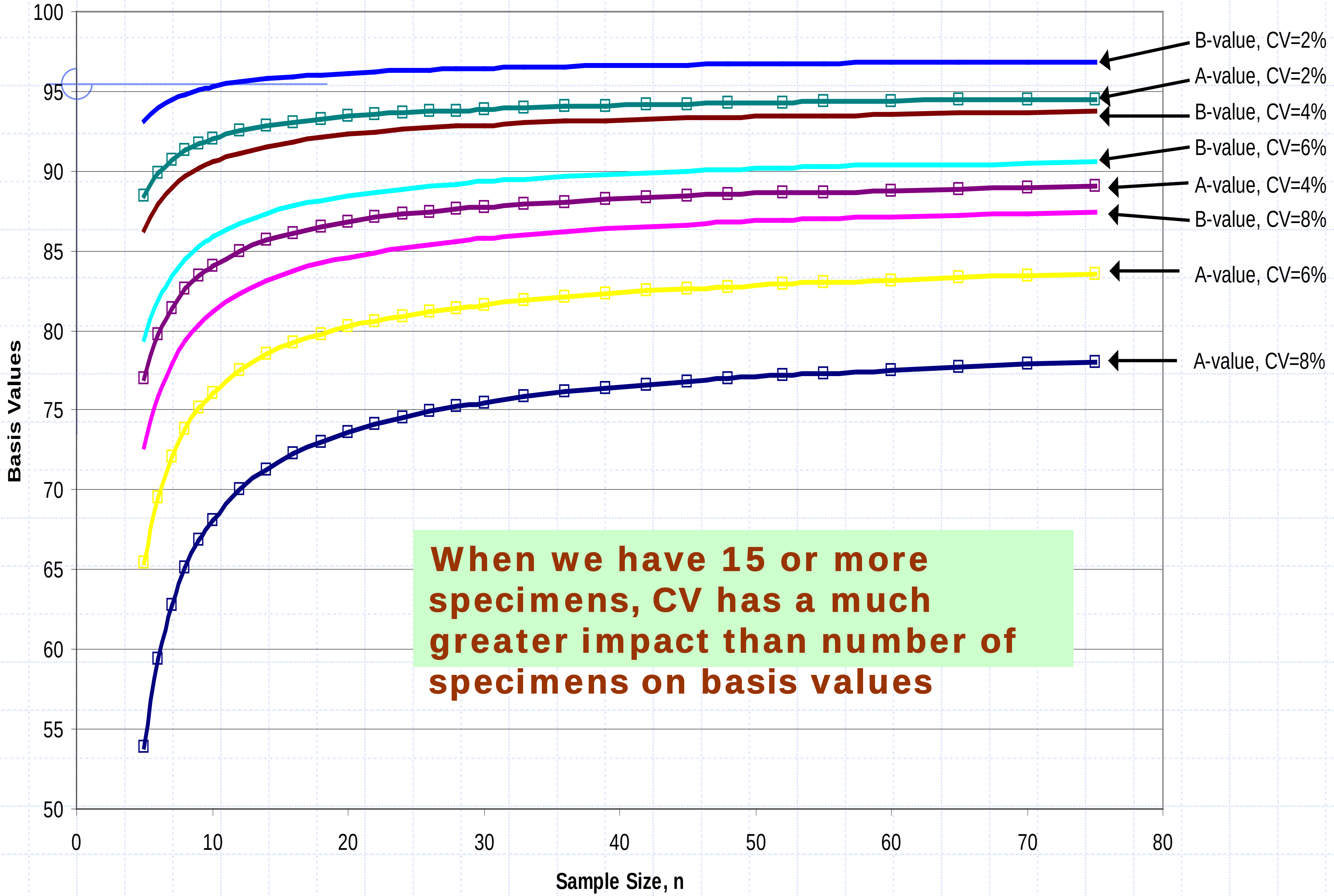
A Common Problem with Qualification

- Often captures a small portion of the true (production material) variability only; as-measured coefficient of variation (CV) is often lower than actual CV
- Qualification/allowable material batches are
  - Usually produced within a short period of time (not in the spring, summer, fall, and winter)
  - Usually produced by one shift (not by first-shift, second-shift, and third-shift)
  - Not all raw materials (e.g. base resin, curing agents, and modifiers/additives/fillers) are of distinct batches
  - Therefore, not representative of actual production material batches
- Therefore, qualification/allowable material batches often produce unrealistically low CV and unconservatively high A- and B-basis values and unrealistically high spec limits
- What should we do to protect our allowables? Answer: Modified CV

As-Measured Data May Not Capture the True (Actual Production) Material Variability



Effects of CV and Sample Size on Basis Values
Mean=100



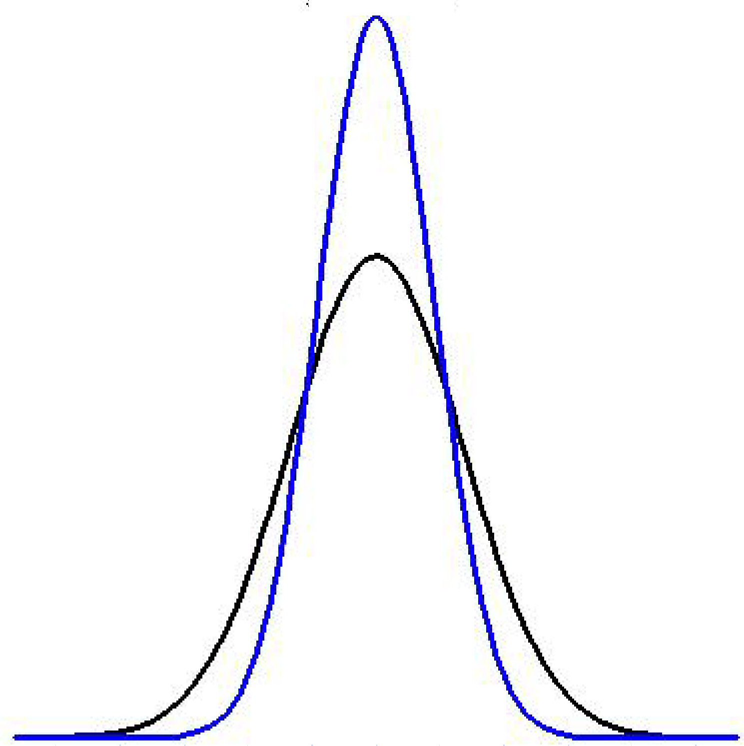
Modified Coefficient of Variation (CV)

- In order to compensate for the smaller variation of the qualification sample, the CV is increased PRIOR to computing the A and B basis values and specification limits
- The main effects of this:
  - Decrease the computed A- and B-basis values
  - Decrease the equivalence and acceptance limits
  - Reduce the need to use engineering judgment per 8.3.10.1 (related to low CV; see yellow pages); *reduce false alarms due to batch-to-batch variability*

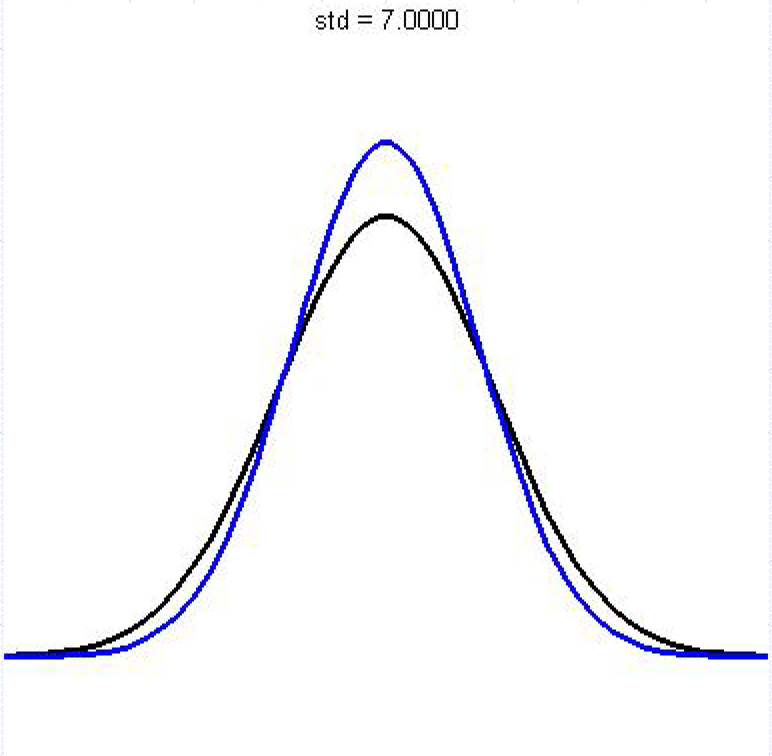
Modification Rules

1. If the CV is below 4%,
   then the modified CV is 6%
2. If the CV is between 4% and 8%,
   then the modified CV = (0.5 * CV) + 4%
3. If the CV is over 8%
   then no modification is made

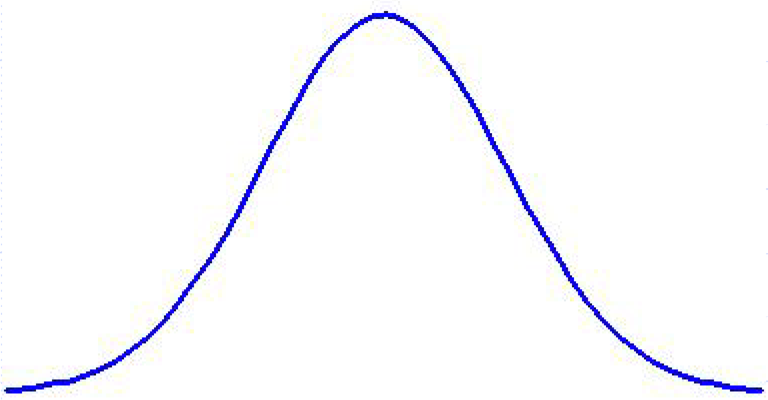
A CV of 4 increases to 6



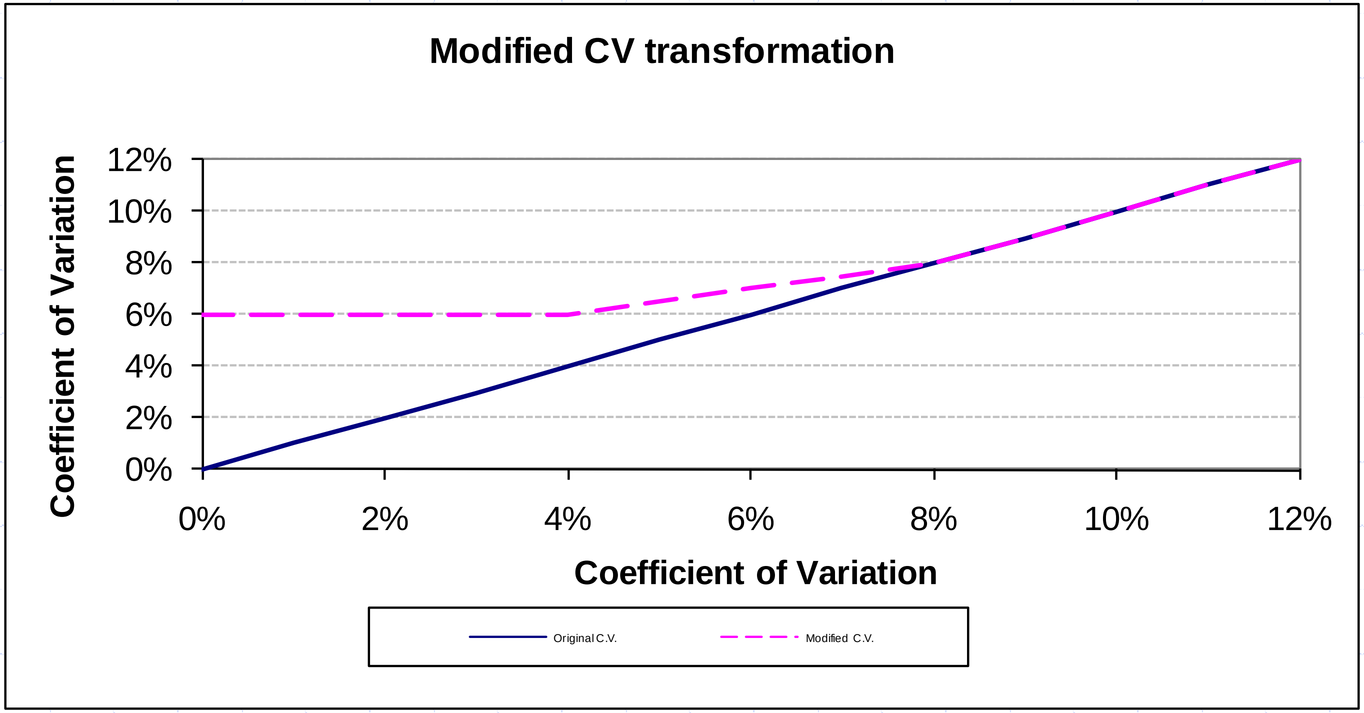
A CV of 6 increases to 7



A CV of 8 doesn't change

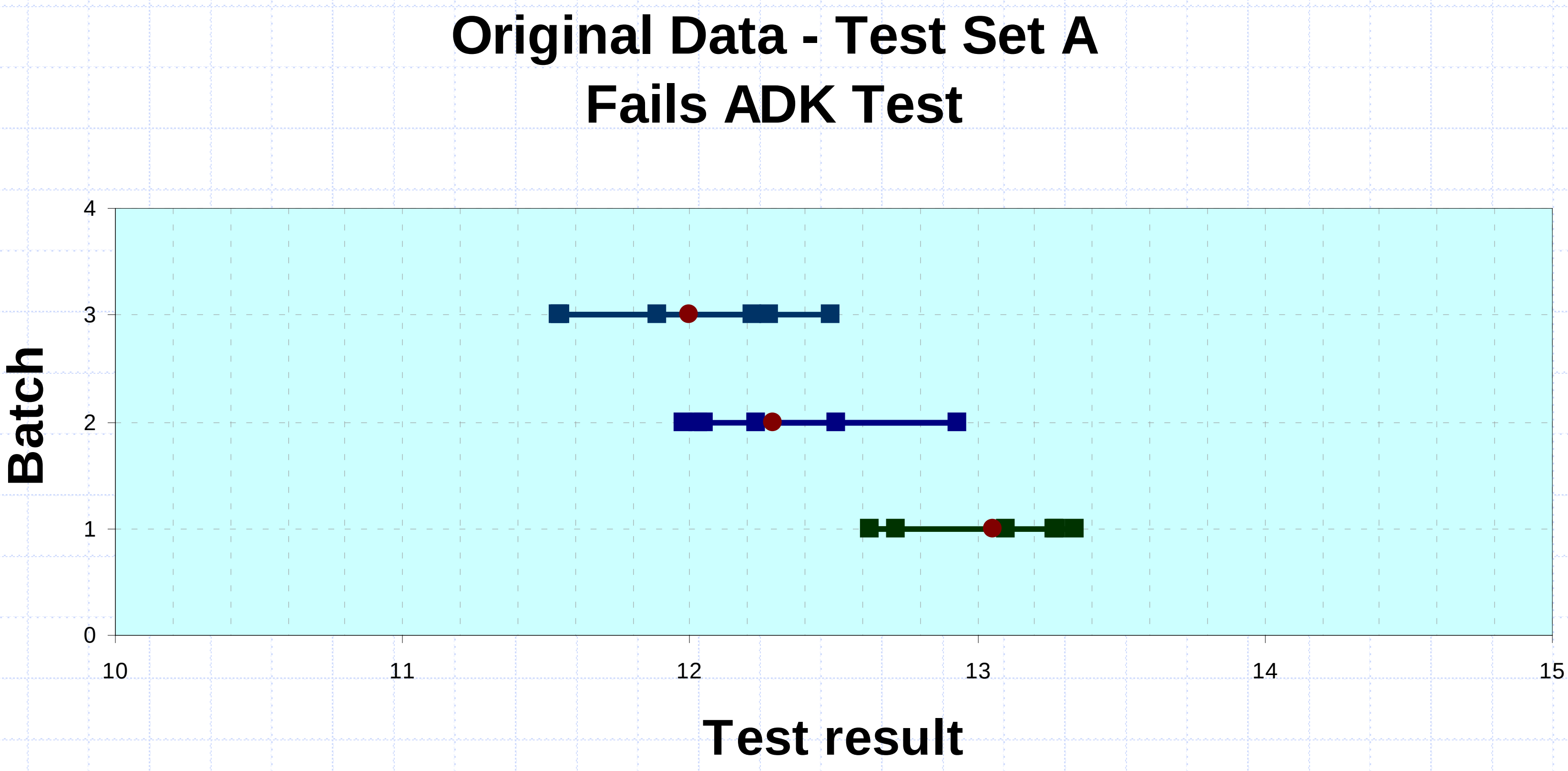


Effect of CV Modification

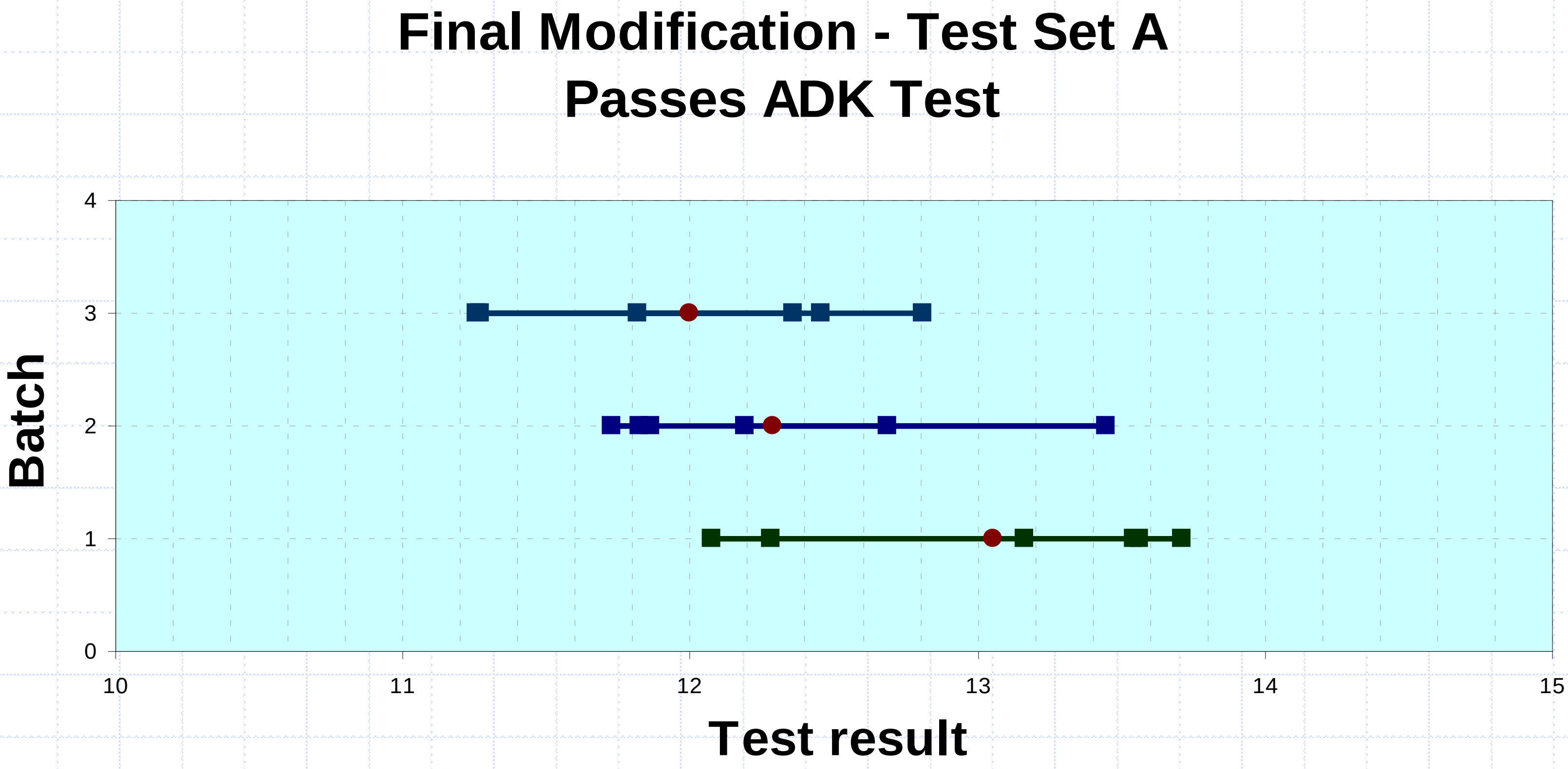


If the CV is 10% or more then the modified CV is 10% for setting specification limits; use as-measured CV for basis value computation

Original Individual Values in Diagnostic Tests

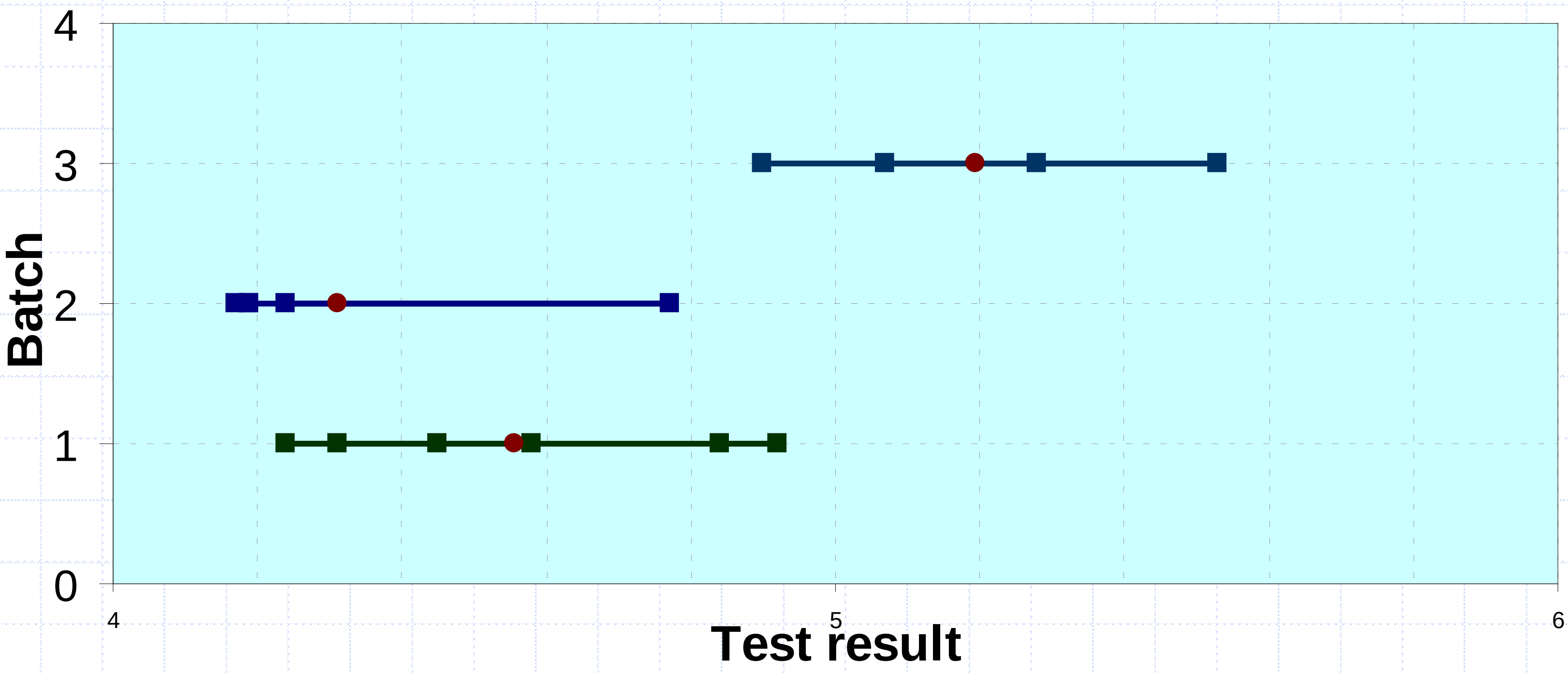


Transformed Individual Values in Diagnostic Tests



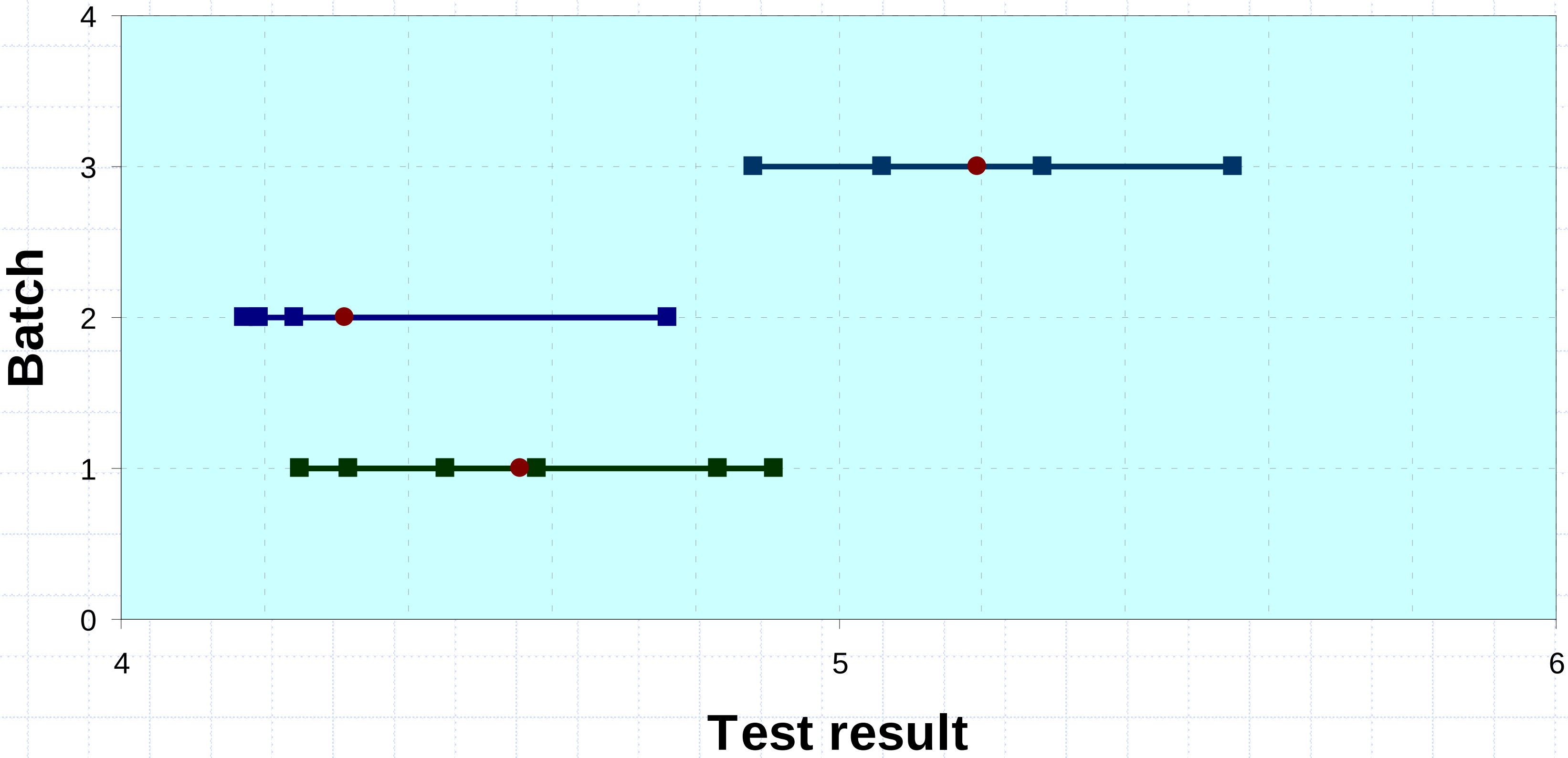
Transformation of Individual values for diagnostic tests

Original Data - Test Set B Fails ADK Test



Transformation of Individual values for diagnostic tests

Final Modification - Test Set B Fails ADK Test



Conclusions About Modified CV

- As-measure CV may not capture the true material property variability, therefore, may result in higher than actual basis values (unconservative)

- Modified CV accounts for sources of variability not present in the qualification material and testing

- Modified CV reduces the need to use engineering judgment per 8.3.10.1 (see yellow pages); *reduces false alarms (due to low CV) of batch-to-batch variability test*

THE LATEST VERSION OF ASAP INCORPORATES MODIFIED CV IN THE STATISTICAL TESTS AND BASIS VALUE COMPUTATIONS